

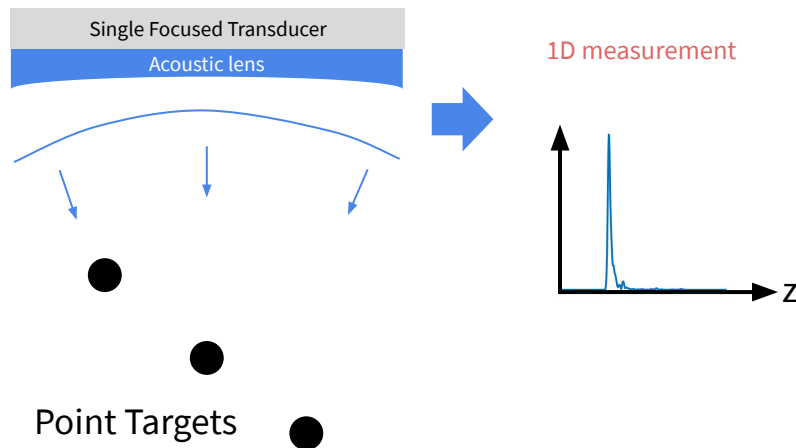
# Single Element Ultrasound Imaging with Compressed Sensing

William Meng  
EE 367 Final Project  
March 17, 2021

# Background

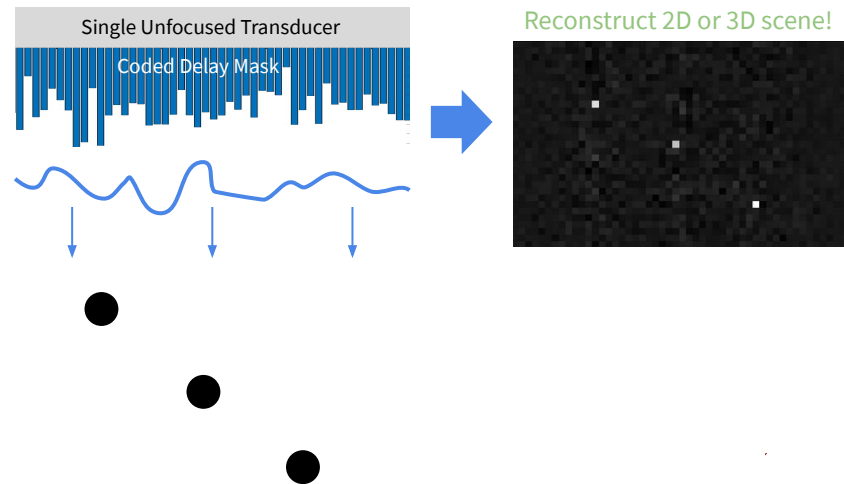
## Single Element Ultrasound Imaging (A-mode)

- Capture time-series data to measure depth information
- Time of arrival indicates depth of target
- No lateral information



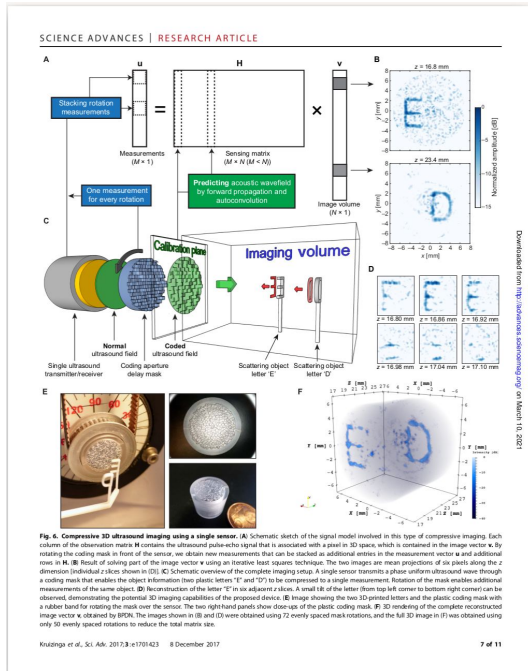
## Single Element Ultrasound Imaging with Compressed Sensing

- Use laterally-varying pseudorandom delay mask to encode lateral information into time-series data, in addition to the depth information already captured.



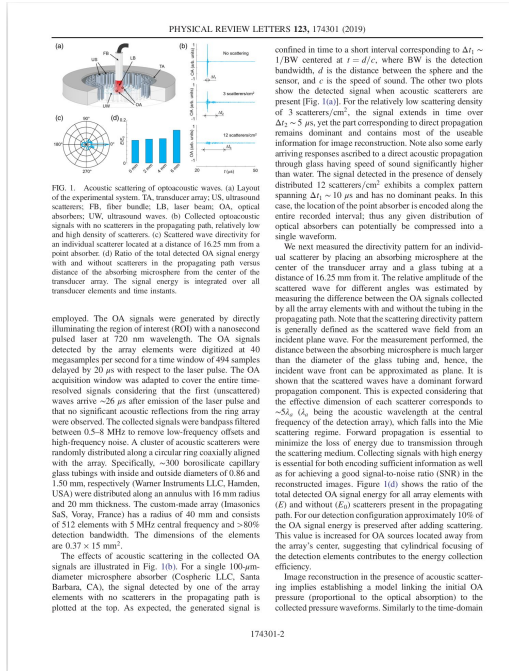
# Related Works

Kruizinga et al: [“Compressive 3D ultrasound imaging using a single sensor”](#)



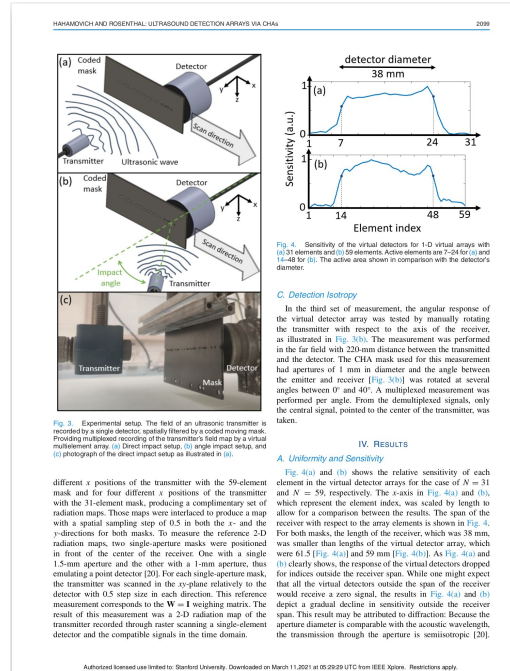
phase/delay mask

Deán-Ben et al: [“Acoustic Scattering Mediated Single Detector Optoacoustic Tomography”](#)



scattering layer

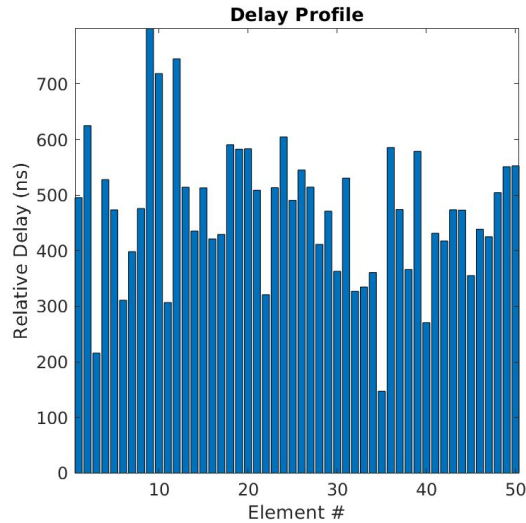
Hahamovich et al: [“Ultrasound Detection Arrays via Coded Hadamard Apertures”](#)



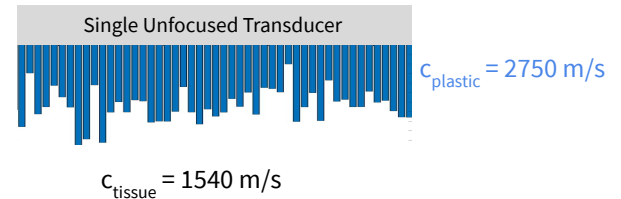
amplitude mask

# Approximating the mask as a delay profile

*Delay profile in simulation*



*Corresponding physical mask*



More complicated to simulate due to heterogeneity.

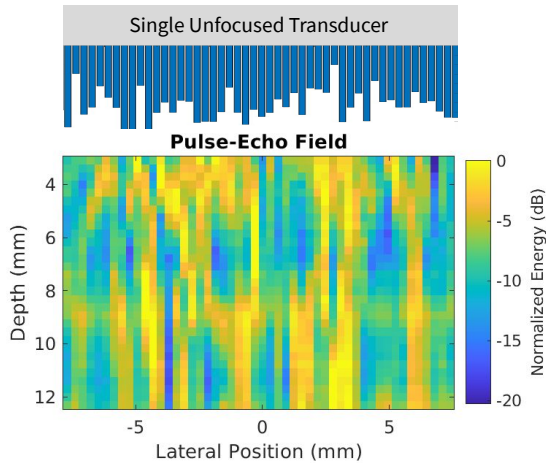
Ignores:

- Reflection at interface
- Refraction at interface
- Wave spreading within the mask

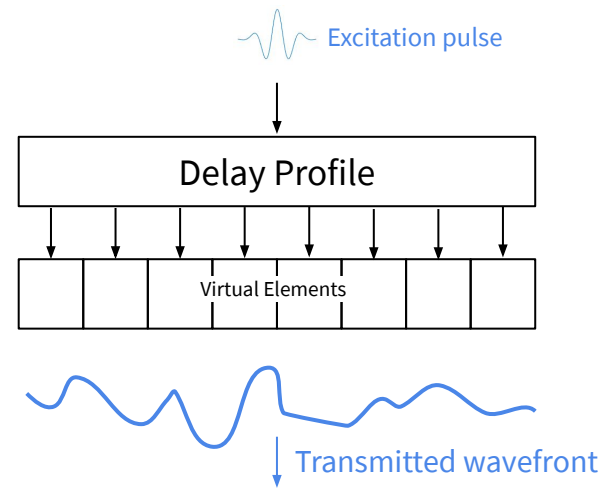
Much easier and faster to simulate!

# Ultrasound Simulation

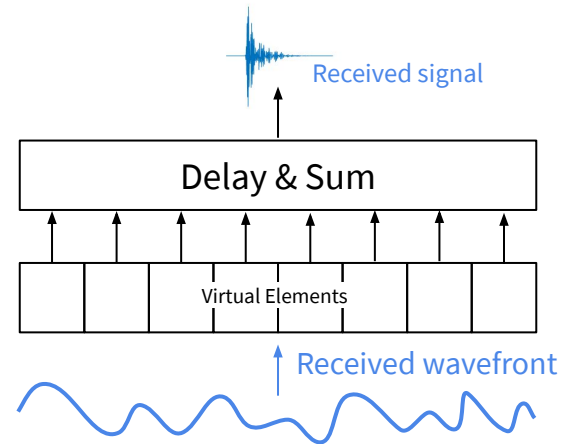
Used *Field II* in Matlab to simulate pulse-echo response:



Tx



Rx



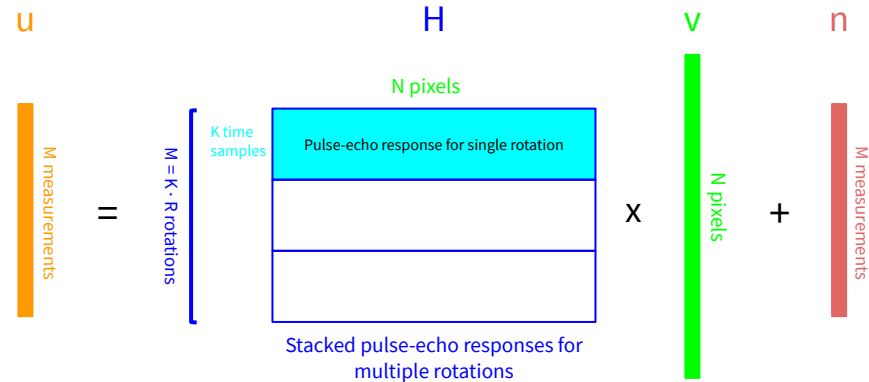
# Image Formation Model

Image formation model:

$$u = Hv + n$$

Where:

- $v$  = ground truth image
  - Size:  $(N, 1)$
- $H$  = image formation matrix
  - Size:  $(M, N)$
- $n$  = additive Gaussian noise
  - Size:  $(M, 1)$
- $u$  = measured data
  - Size:  $(M, 1)$



Dimensions:

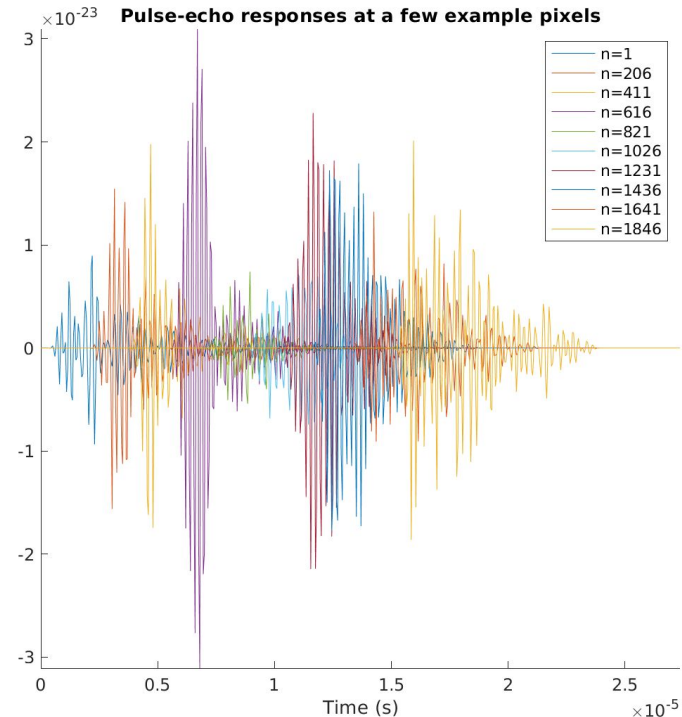
- $N$  = number of pixels in image
- $M = RK$  = number of measurements
  - $K$  = number of time samples in measured signal for each rotation
  - $R$  = number of rotations

# Example of some pulse-echo waveforms

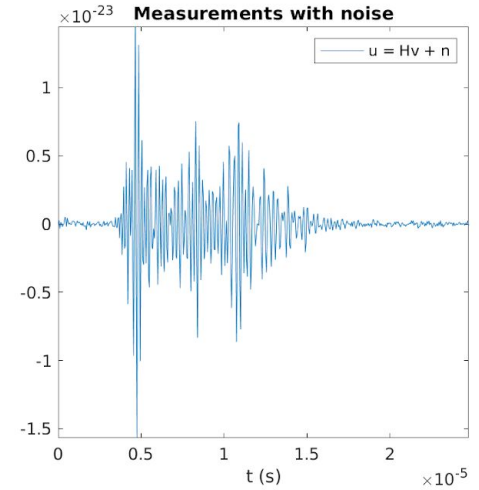
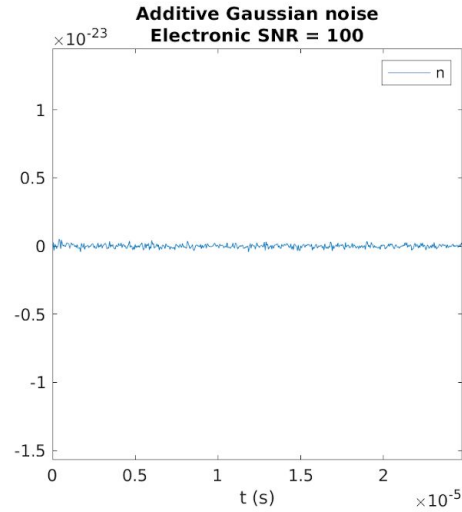
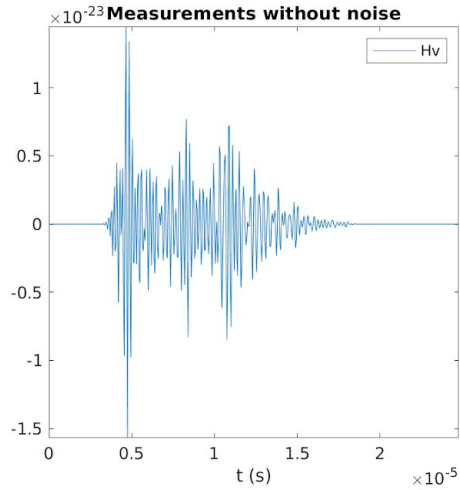
Each waveform shown here is the time-series data that represents the pulse-echo response for a pixel  $n$  in the field of view.

In the single rotation case, each column of  $H$  is just one of these waveforms.

In the multi-rotation case, each column of  $H$  is formed by concatenating the waveforms from each rotation.



# Example Measurement





# Reconstruction Algorithms

## *Least Norm Solution*

Problem:  $\min_{\hat{v}} \|H\hat{v} - u\|_2^2$

Solution:  $\hat{v} = H^T(HH^T)^{-1}u$

Implemented with:

- Preconditioned Conjugate Gradient (PCG)
- Moore-Penrose Pseudo-inverse

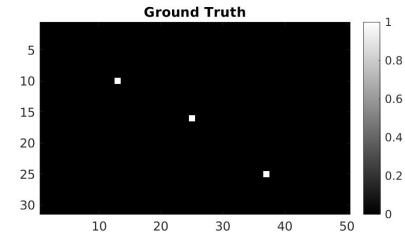
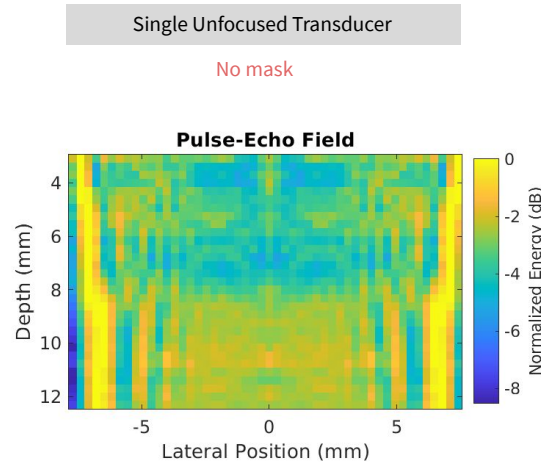
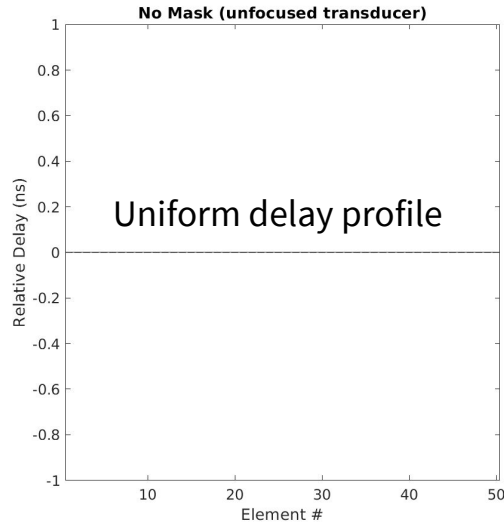
## *ADMM*

See EE 367 Lecture 11 notes for more details.

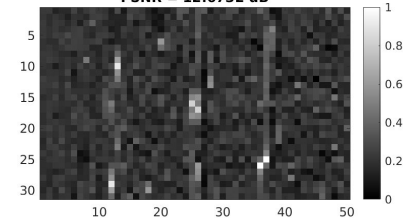
# Image Reconstruction (no mask)

The pulse-echo field with no mask actually does have some spatiotemporal diversity due to the near-field interference pattern of an unfocused transducer.

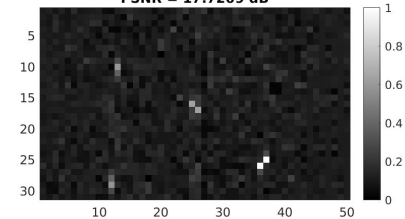
However, there are substantial artifacts in the reconstructed image due to the large amount of symmetry and self-similarity in the pulse-echo field. You can see erroneous “double point” targets in the reconstructed images, as well as a lot of background noise.



**Least Norm (PCG) Solution**  
maxitersCG = 1000, tolCG = 1.20525e-32  
Runtime = 1.05212 s  
PSNR = 12.6732 dB



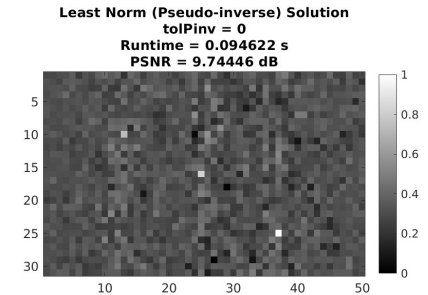
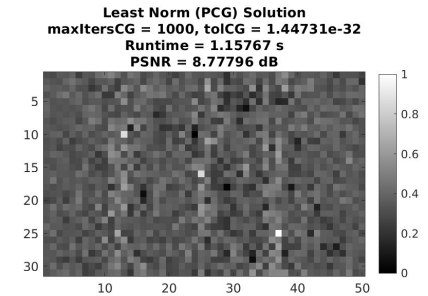
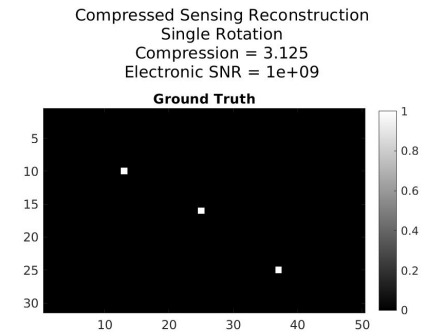
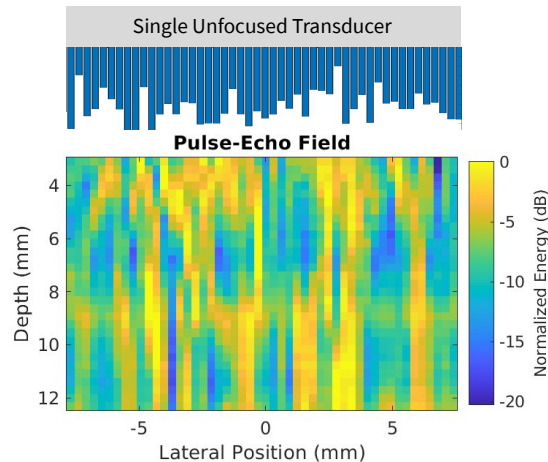
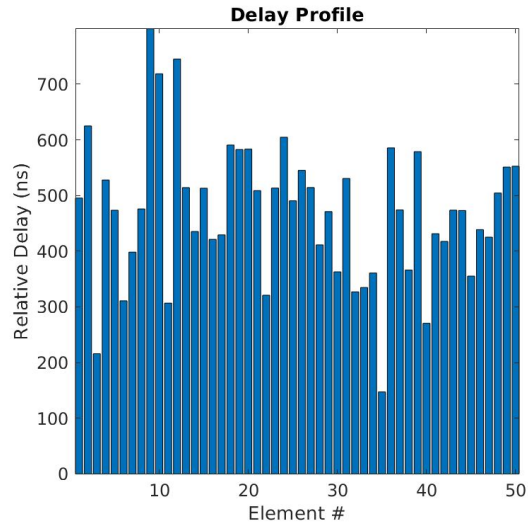
**Least Norm (Pseudo-inverse) Solution**  
tolPinv = 0  
Runtime = 0.085966 s  
PSNR = 17.7269 dB



# Image Reconstruction (single rotation)

The pulse-echo field with a single rotation of the mask is highly aberrated and has little symmetry.

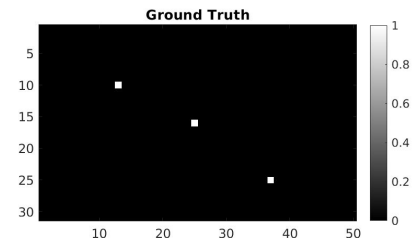
The reconstructed image resolves the 3 point targets with high spatial accuracy, but there are artifacts that appear as background noise, so the PSNR is actually worse than the “no mask” case.



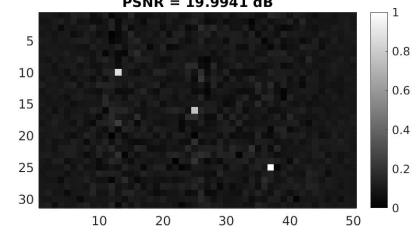
# Image Reconstruction (4 rotations)

Each rotation produces a completely different pulse-echo pattern, which captures more information about the scene.

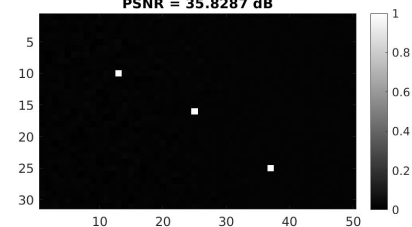
Very good image reconstruction quality, as indicated by high PSNR. Pseudo-inverse solution looks pretty much perfect.



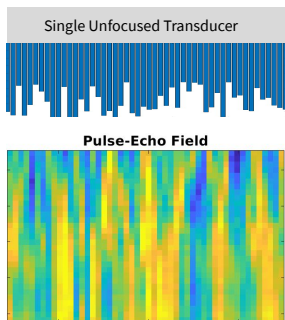
**Least Norm (PCG) Solution**  
maxItersCG = 1000, tolCG = 3.98246e-32  
Runtime = 4.05171 s  
PSNR = 19.9941 dB



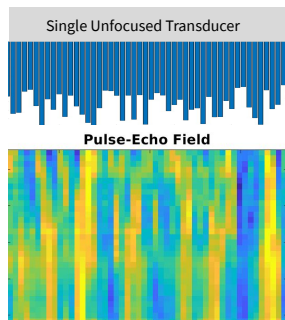
**Least Norm (Pseudo-inverse) Solution**  
tolPInv = 0  
Runtime = 4.46175 s  
PSNR = 35.8287 dB



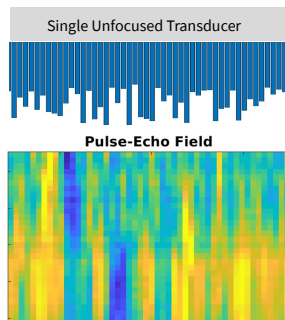
*Rotation 1*



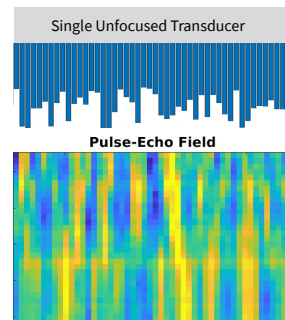
*Rotation 2*



*Rotation 3*

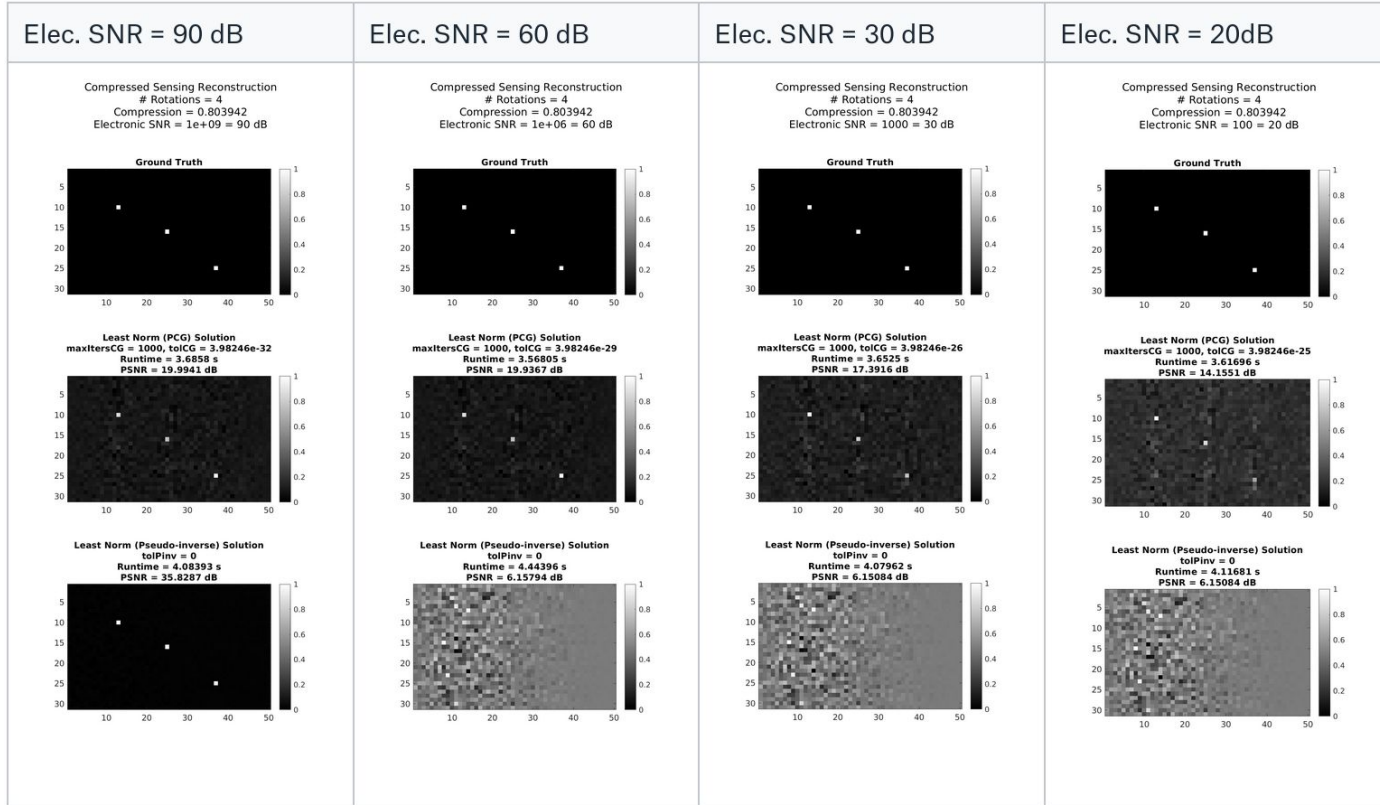


*Rotation 4*



# Impact of electronic SNR

(R=4 in all cases)



PCG image degrades gradually with electronic SNR.

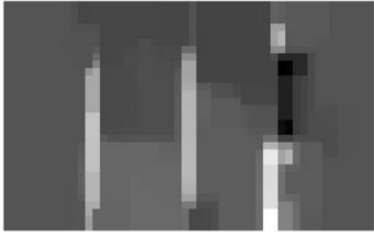

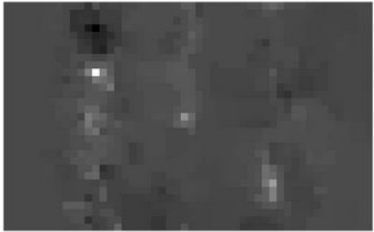

Pseudo-inverse fails unless you have very high electronic SNR.

# ADMM with anisotropic TV regularization

ADMM ended up producing worse results than the Least Norm solutions.

Maybe there is an issue with the parameters I chose?

Or perhaps the TV regularizer doesn't work well for the scene with point targets?

No Mask	1 Rotation	4 Rotations	10 Rotations
<p><math>\lambda = 0.01, \rho = 10</math> PSNR = 8.44859 dB Runtime = 9.7165 s</p> 	<p><math>\lambda = 0.01, \rho = 10</math> PSNR = 3.91739 dB Runtime = 8.63383 s</p> 	<p><math>\lambda = 0.01, \rho = 10</math> PSNR = 10.684 dB Runtime = 10.2342 s</p> 	<p><math>\lambda = 0.01, \rho = 10</math> PSNR = 16.2542 dB Runtime = 14.8637 s</p> 

## Further work

- Image more complicated scenes.
- Use real-world ultrasound data instead of idealized synthetic data.
- Incremental reconstruction of a dynamic scene using a Kalman filter
  - Instead of simply taking an ensemble average of each individual reconstruction, iteratively combine the previous reconstruction with the new one based on statistical properties.
- Reconstruct a 3D volume
- Parallelize the reconstruction algorithm to run on a GPU
- Use a specifically designed mask like a Coded Hadamard Aperture

# References

[1] P. Kruizinga, et al, “Compressive 3D ultrasound imaging using a single sensor”, Science Advances, Vol. 3, No. 12, December 2017.  
<https://advances.sciencemag.org/content/3/12/e1701423>

[2] E. Hahamovich, A. Rosenthal, “Ultrasound Detection Arrays Via Coded Hadamard Apertures”, IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, Vol. 67, Issue 10, Oct. 2020.  
<https://ieeexplore.ieee.org/document/9090912>

[3] X. Luís Deán-Ben, et al, “Acoustic Scattering Mediated Single Detector Optoacoustic Tomography”, Physical Review Letters, Vol. 123, Iss. 17, 25 October 2019.  
<https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.123.174301>

Code acknowledgments:

- Ultrasound simulation was performed using the [Field II](#) library in Matlab, with code adapted from the RAD 235 workshops.
- PCG and ADMM code was adapted from the EE 367 homework.



Thank you!

Please email me at [wlmeng@stanford.edu](mailto:wlmeng@stanford.edu) if you have any questions!

Poster Version

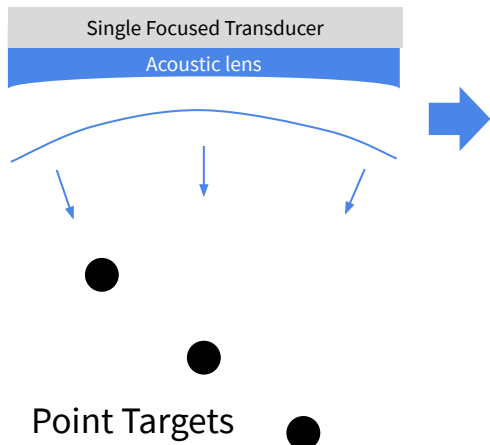


## Figures for Paper

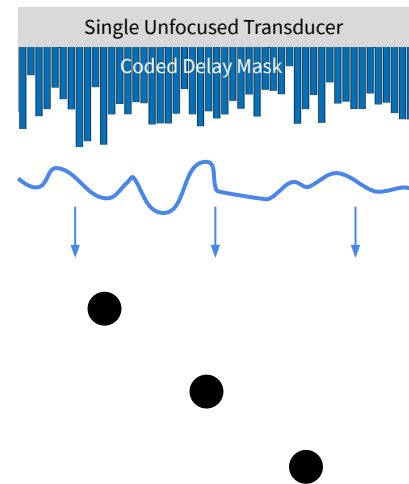
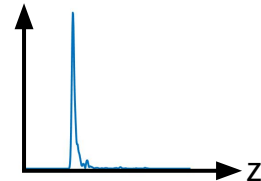


## Fig. 1

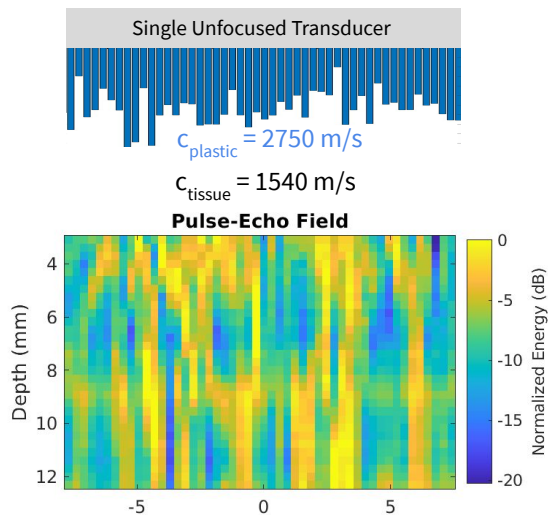
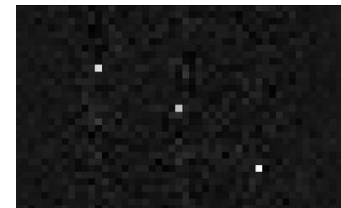
- a) A-mode
- b) compressed sensing
- c) physical mask  $\rightarrow$  delay profile
- d) block diagram with virtual elements



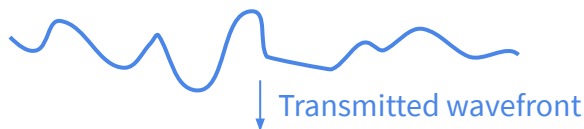
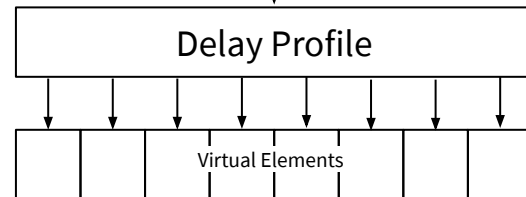
1D measurement



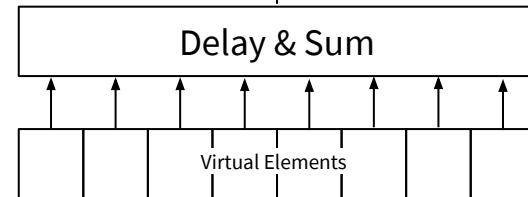
Reconstruct 2D or 3D scene!



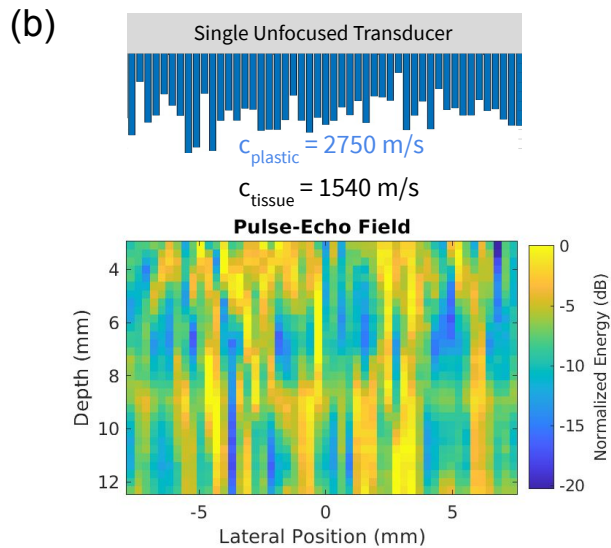
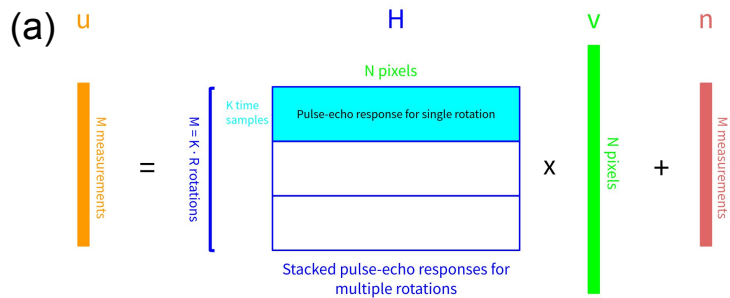
Tx



Rx



Stanford University



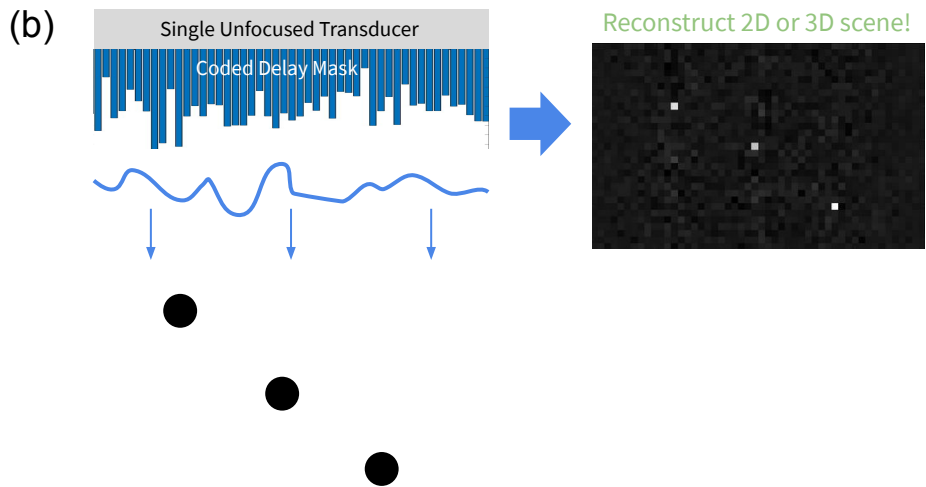
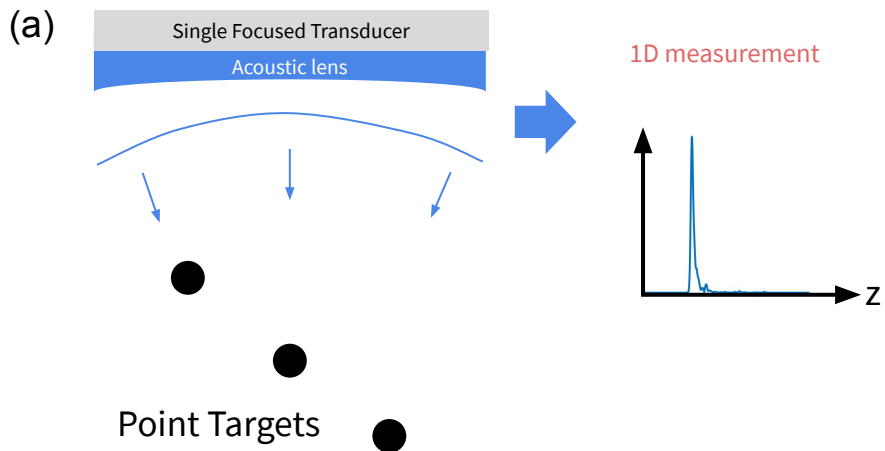


Fig. 2

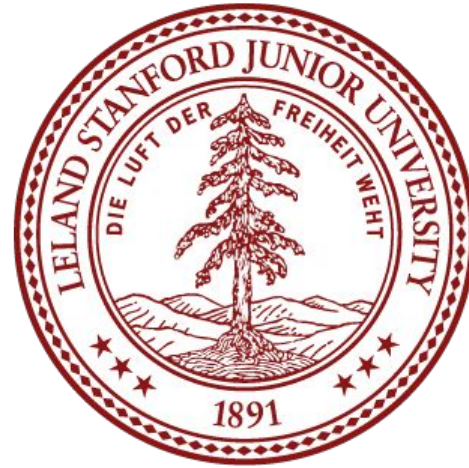


Fig. 3

Fig. 4

Fig. 5

Archived



# Simulation Approach

Approximate delay mask as a near-field phase mask.

- Ignore amplitude
- Ignore reflection & refraction effects

In essence, each element in the mask will only affect the local delay on the transducer.

By using this approximation, we avoid simulating the wave propagation in the plastic material (which would require a finer grid size).

Instead, we perform the simulation in a homogeneous medium, which is computationally more efficient.

# Physical Representation vs. Simulation

## Physical

uniform wavefront from  
transducer, aberrations produced  
by propagation through delay  
mask

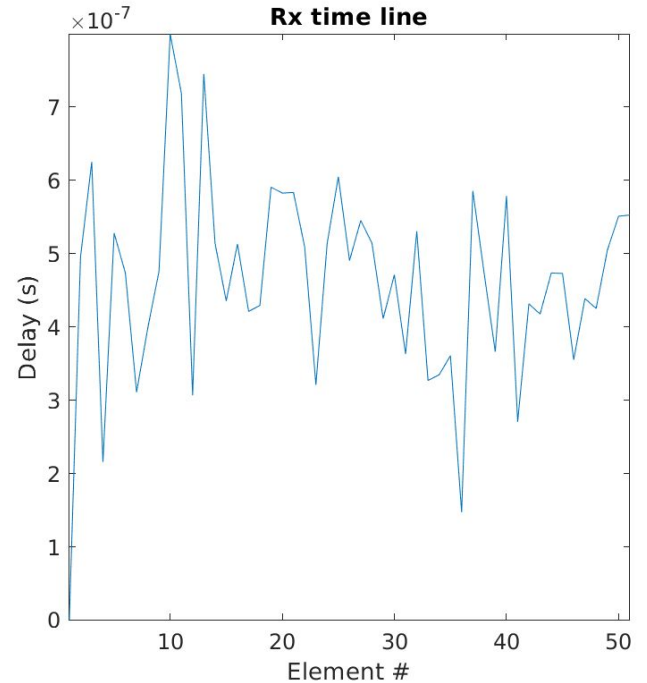
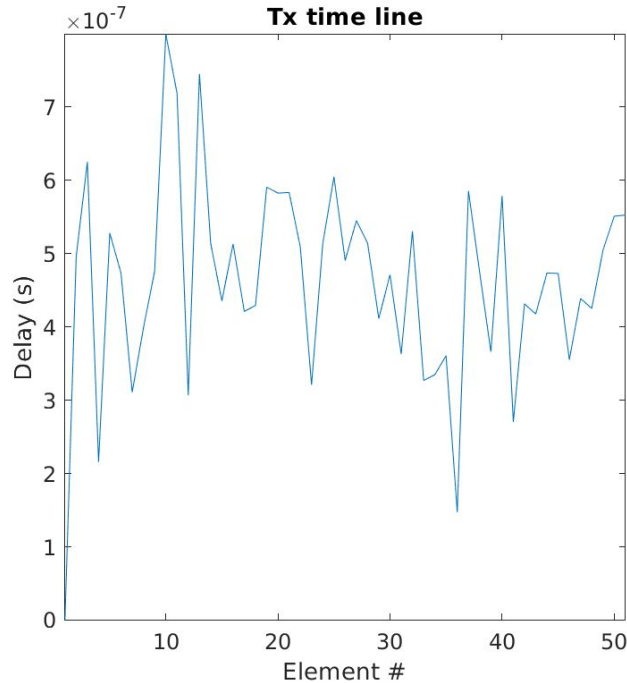
## Simulation

aberrations produced directly by  
delays applied to each element in  
transducer array  
summing across all elements

# Localized delays

In the simulation, we are defining a multi-element array for the sole purpose of emulating a physical delay mask. The “single sensor measurement” can be attained by averaging the signal measured by all the elements.

Tx and Rx timelines



# True Image

True image

